Here are some problems to get you more practice gluing. Let k be a field.

Problem 1 Let $X_1 = \operatorname{Spec} k[t_1, u_1, u_1^{-1}]$ and let $X_2 = \operatorname{Spec} k[t_2, u_2, u_2^{-1}]$. Glue $D(t_1) \subset X_1$ to $D(t_2) \subset X_2$ by identifying $(t_1, u_1) \sim (t_2, t_2 u_2)$; call the result X.

(a) Show that the ring of regular functions on X is isomorphic to $k[x, y]$.

(b) Show that X is isomorphic to $\operatorname{Spec} k[x, y] \setminus V(x, y)$.

Problem 2 Let $Y_1 = \operatorname{Spec} k[t_1, v_1]$ and $Y_2 = \operatorname{Spec} k[t_2, v_2]$. Glue $D(t_1) \subset Y_1$ to $D(t_2) \subset Y_2$ by identifying $(t_1, v_1) \sim (t_2, v_2 + t_2^{-1})$; call the result Y.

(a) Define the following functions on Y_1: $t = t_1$, $w = t_1 v_1$ and $x = (t_1 v_1 - 1) v_1$. Show that these functions extend to all of Y.

(b) Show that Y is isomorphic to $\operatorname{Spec} k[t, w, x] / \langle tx - w(w-1) \rangle$.

Problem 3 This is meant to be easy.

(a) Show that the schemes X and Y above are separated.

Let L be the "line with a double point" over k: Two copies of $\mathbb{A}^1_k$ glued everywhere except at the origin.

(b) Show that X and Y have surjections to L.